

(PITHA 00/13, HEP-PH/0007256, JULY 2000)

QCD FACTORIZATION FOR $B \rightarrow \pi K$ DECAYS

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We examine some consequences of the QCD factorization approach to non-leptonic B decays into πK and $\pi\pi$ final states, including a set of enhanced power corrections. Among the robust predictions of the approach we find small strong-interaction phases (with one notable exception) and a pattern of CP-averaged branching fractions, which in some cases differ significantly from the current central values reported by the CLEO Collaboration.

1 Introduction

The observation of B decays into πK and $\pi\pi$ final states has resulted in a large amount of theoretical and phenomenological work that attempts to interpret these observations in terms of the factorization approximation (FA), or in terms of general parameterizations of the decay amplitudes. A detailed understanding of these amplitudes would help us to pin down the value of the CKM angle γ using only data on CP-averaged branching fractions. Theoretical work on the heavy-quark limit has justified the FA as a useful starting point^{1,2}, but predicts important and computable corrections. Here we discuss the most important consequences of this approach for the πK and $\pi\pi$ final states.^a

To leading order in an expansion in powers of Λ_{QCD}/m_b , the $B \rightarrow \pi K$ matrix elements obey the factorization formula

$$\langle \pi K | Q_i | B \rangle = f_+^{B \rightarrow \pi}(0) f_K T_{K,i}^I * \Phi_K$$

$$+ f_+^{B \rightarrow K}(0) f_\pi T_{\pi,i}^I * \Phi_\pi \quad (1)$$

$$+ f_B f_K f_\pi T_i^{\text{II}} * \Phi_B * \Phi_K * \Phi_\pi,$$

where Q_i is an operator in the weak effective Hamiltonian, $f_+^{B \rightarrow M}(0)$ are semi-leptonic form factors of a vector current evaluated at $q^2 = 0$, Φ_M are leading-twist light-cone distribution amplitudes, and the $*$ -products imply an integration over the light-cone momentum fractions of the constituent quarks inside the mesons. When the hard-scattering functions T are evaluated to order α_s^0 , Eq. (1) reduces to the conventional FA. The subsequent results are based on kernels including all corrections of order α_s . A detailed justification of (1) is given in Ref. ². Compared to our previous discussion of $\pi\pi$ final states¹ the present analysis incorporates three new ingredients:

- i) the matrix elements of electroweak (EW) penguin operators (for πK modes);
- ii) hard-scattering kernels for general, asymmetric light-cone distributions;
- iii) the complete set of “chirally enhanced” $1/m_b$ corrections.¹

^aContribution to ICHEP2000, July 27 - August 2, Osaka, Japan (talk presented by M. Beneke).

The second and third items have not been considered in other³ generalizations of Ref. ¹ to the πK final states. The third one, in particular, is essential for estimating some of the theoretical uncertainties of the approach.

We now briefly present the input to our calculations. Following Ref. ¹, we obtained the coefficients $a_i(\pi K)$ of the effective factorized transition operator defined analogously to the case of $\pi\pi$ final states, but augmented by coefficients $a_{7-10}(\pi K)$ related to EW penguin operators and electro-magnetic penguin contractions of current-current and QCD penguin operators. A sensible implementation of QCD corrections to EW penguin matrix elements implies that one departs from the usual renormalization-group counting, in which the initial condition for EW penguin coefficients is treated as a next-to-leading order (NLO) effect. Our NLO initial condition hence includes the α_s corrections computed in Ref. ⁴.

Chirally enhanced corrections arise from twist-3 two-particle light-cone distribution amplitudes, whose normalization involves the quark condensate. The relevant parameter, $2\mu_\pi/m_b = -4\langle\bar{q}q\rangle/(f_\pi^2 m_b)$, is formally of order Λ_{QCD}/m_b , but large numerically. The coefficients a_6 and a_8 are multiplied by this parameter. There are also additional chirally enhanced corrections to the spectator-interaction term in (1), which turn out to be the more important effect. In both cases, these corrections involve logarithmically divergent integrals, which violate factorization. For instance, for matrix elements of $V-A$ operators the hard spectator interaction is now proportional to $(\bar{u} \equiv 1 - u)$

$$\int_0^1 \frac{du}{\bar{u}} \frac{dv}{\bar{v}} \Phi_K(u) \left(\Phi_\pi(v) + \frac{2\mu_\pi}{m_b} \frac{\bar{u}}{u} \right)$$

when the spectator quark goes to the pion. (Here we used that the twist-3 distribution amplitudes can be taken to be the asymptotic ones when one neglects twist-3 corrections without the chiral enhancement.) The divergence of the v -integral in the second term as $\bar{v} \rightarrow 0$ implies that it is dominated

by soft gluon exchange between the spectator quark and the quarks that form the kaon. We therefore treat the divergent integral $X = \int_0^1 (dv/\bar{v})$ as an unknown parameter (different for the penguin and hard scattering contributions), which may in principle be complex owing to soft rescattering in higher orders. In our numerical analysis we set $X = \ln(m_B/0.35 \text{ GeV}) + r$, where r is chosen randomly inside a circle in the complex plane of radius 3 (“realistic”) or 6 (“conservative”). Our results also depend on the B -meson parameter¹ λ_B , which we vary between 0.2 and 0.5 GeV. Finally, there is in some cases a non-negligible dependence of the coefficients $a_i(\pi K)$ on the renormalization scale, which we vary between $m_b/2$ and $2m_b$.

2 Results

We take $|V_{ub}/V_{cb}| = 0.085$ and $m_s(2 \text{ GeV}) = 110 \text{ MeV}$ as fixed input, noting that ultimately the ratio $|V_{ub}/V_{cb}|$, along with the CP-violating phase $\gamma = \arg(V_{ub}^*)$, might be extracted from a simultaneous fit to the $B \rightarrow \pi K$ and $B \rightarrow \pi\pi$ decay rates.

2.1 $SU(3)$ breaking

Bounds^{5,6} on the CKM angle γ derived from ratios of πK branching fractions, as well as the determination of γ using the method of Ref. ⁷, rely on an estimate of $SU(3)$ flavour-symmetry violations. We find that “non-factorizable” $SU(3)$ -breaking effects (i.e., effects not accounted for by the different decay constants and form factors of pions and kaons in the conventional FA) do not exceed a few percent at leading power.

2.2 Amplitude parameters

The approach discussed here allows us to obtain the decay amplitudes for the $\pi\pi$ and πK final states in terms of the form factors and the light-cone distribution amplitudes. The $\pi^0\pi^0$ final state is very poorly predicted and will not be discussed here. We write

$$\mathcal{A}(B^0 \rightarrow \pi^+\pi^-) = T[e^{i\gamma} + (P/T)_{\pi\pi}]$$

Table 1. Parameters for the $B \rightarrow \pi K$ amplitudes as defined in (2), for conservative variation of all input parameters (see text).

	Range, NLO	LO
$-\varepsilon_a e^{i\eta}$	$(0.017-0.020) e^{i[13,21]^\circ}$	0.02
$\varepsilon_{3/2} e^{i\phi}$	$(0.20-0.38) e^{i[-30,7]^\circ}$	0.36
$q e^{i\omega}$	$(0.53-0.63) e^{i[-7,3]^\circ}$	0.64
$\varepsilon_T e^{i\phi_T}$	$(0.20-0.29) e^{i[-19,3]^\circ}$	0.33
$q_C e^{i\omega_C}$	$(0.00-0.22) e^{i[-180,180]^\circ}$	0.06
$(P/T)_{\pi\pi}$	$(0.19-0.29) e^{i[-1,23]^\circ}$	0.16

and parameterize the πK amplitudes by⁶

$$\begin{aligned}
\mathcal{A}(B^+ \rightarrow \pi^+ K^0) &= P(1 - \varepsilon_a e^{i\eta} e^{i\gamma}), \\
-\sqrt{2} \mathcal{A}(B^+ \rightarrow \pi^0 K^+) &= P \left[1 - \varepsilon_a e^{i\eta} e^{i\gamma} \right. \\
&\quad \left. - \varepsilon_{3/2} e^{i\phi} (e^{i\gamma} - q e^{i\omega}) \right], \quad (2) \\
-\mathcal{A}(B^0 \rightarrow \pi^- K^+) &= P \left[1 - \varepsilon_a e^{i\eta} e^{i\gamma} \right. \\
&\quad \left. - \varepsilon_T e^{i\phi_T} (e^{i\gamma} - q_C e^{i\omega_C}) \right],
\end{aligned}$$

and $\sqrt{2} \mathcal{A}(B^0 \rightarrow \pi^0 K^0) = \mathcal{A}(B^+ \rightarrow \pi^+ K^0) + \sqrt{2} \mathcal{A}(B^+ \rightarrow \pi^0 K^+) - \mathcal{A}(B^0 \rightarrow \pi^- K^+)$. Table 1 summarizes the numerical values for the amplitude parameters for the conservative variation of X , and variation of the other parameters as explained above. The LO results correspond to the conventional FA at the fixed scale $\mu = m_b$. They are strongly scale dependent. In comparison, the scale-dependence of the NLO result is small, with the exception of $q_C e^{i\omega_C}$. One must keep in mind that the ranges may overestimate the true uncertainty, since the parameter X may ultimately be constrained from a subset of branching fractions. This is true in particular for the quantity $\varepsilon_{3/2}$ in Table 1, which can be extracted from data.⁶

2.3 Ratios of CP-averaged rates

Since the form factor $f_+(0)$ is not well known, we consider here only ratios of CP-averaged branching ratios, discarding the $\pi^0 \pi^0$ final state. We display these as functions of the CKM angle γ in Fig. 1.

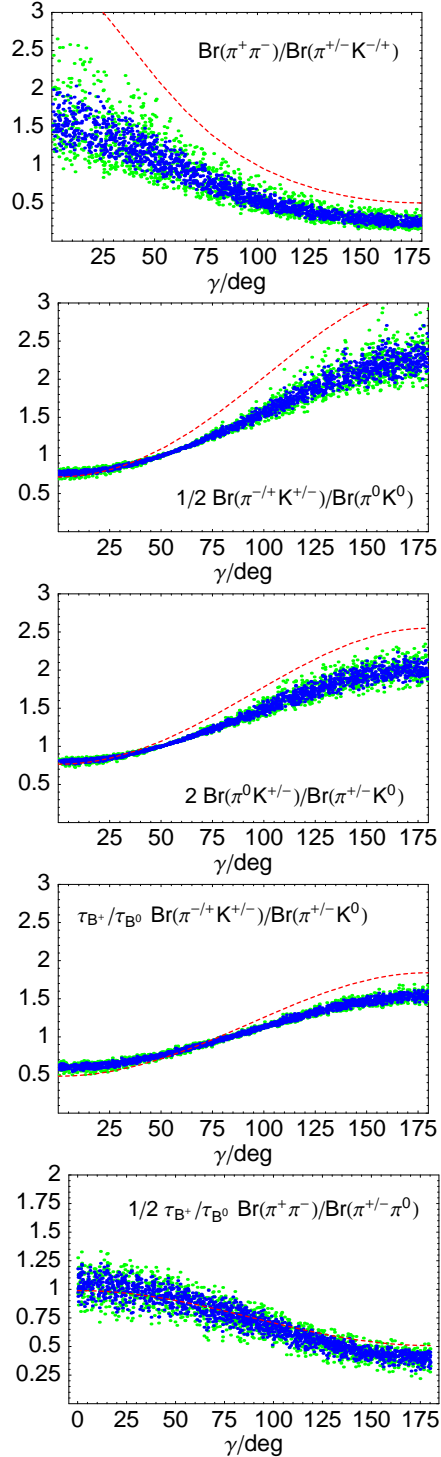


Figure 1. Ratios of CP-averaged $B \rightarrow \pi K$ and $\pi\pi$ decay rates. The scattered points cover a realistic (dark) and conservative (light) variation of input parameters. The dashed curve is the LO result, corresponding to conventional factorization.

Table 1 shows that the corrections with respect to the conventional FA are significant (and important to reduce the renormalization-scale dependence). Despite this fact, the *qualitative* pattern that emerges for the set of πK and $\pi\pi$ decay modes is similar to that in conventional factorization. In particular, the penguin-tree interference is constructive (destructive) in $B \rightarrow \pi^+\pi^-$ ($B \rightarrow \pi^-K^+$) decays if $\gamma < 90^\circ$. Taking the currently favoured range $\gamma = (60 \pm 20)^\circ$, we find the following robust predictions:

$$\begin{aligned} \frac{\text{Br}(\pi^+\pi^-)}{\text{Br}(\pi^\mp K^\pm)} &= 0.5\text{--}1.9 \quad [0.25 \pm 0.10] \\ \frac{\text{Br}(\pi^\mp K^\pm)}{2\text{Br}(\pi^0 K^0)} &= 0.9\text{--}1.4 \quad [0.59 \pm 0.27] \\ \frac{2\text{Br}(\pi^0 K^\pm)}{\text{Br}(\pi^\pm K^0)} &= 0.9\text{--}1.3 \quad [1.27 \pm 0.47] \\ \frac{\tau_{B^+}}{\tau_{B^0}} \frac{\text{Br}(\pi^\mp K^\pm)}{\text{Br}(\pi^\pm K^0)} &= 0.6\text{--}1.0 \quad [1.00 \pm 0.30] \end{aligned}$$

The first ratio is in striking disagreement with current CLEO data⁹ (square brackets). The near equality of the second and third ratios is a result of isospin symmetry.⁶ We find $\text{Br}(B \rightarrow \pi^0 K^0) = (4.5 \pm 2.5) \times 10^{-6} (V_{cb}/0.039)^2 (f_+^{B \rightarrow \pi}(0)/0.3)^2$ almost independently of γ . This is three times smaller than the central value reported by CLEO.

2.4 CP asymmetry in $B \rightarrow \pi^+\pi^-$ decay

The stability of the prediction for the $\pi^+\pi^-$ amplitude suggests that the CKM angle α can be extracted from the time-dependent mixing-induced CP asymmetry in this decay mode, without using isospin analysis. Fig. 2 displays the coefficient S of $-\sin(\Delta M_{B_d} t)$ as a function of $\sin(2\alpha)$ for $\sin(2\beta) = 0.75$, which may be compared with the result in Ref. ⁸. For some values of S there is a two-fold ambiguity (assuming all angles are between 0° and 180°). A consistency check of the approach could be obtained, in principle, from the coefficient of the $\cos(\Delta m_{B_d} t)$ term.

3 Conclusions

We have examined some of the consequences of the QCD factorization approach to B de-

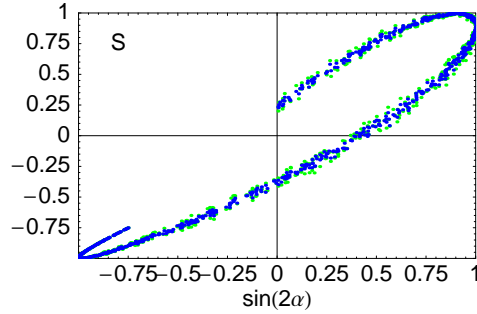


Figure 2. Mixing-induced CP asymmetry in $B \rightarrow \pi^+\pi^-$ decays. The lower band refers to values $45^\circ < \alpha < 135^\circ$, the upper one to $\alpha < 45^\circ$ (right) or $\alpha > 135^\circ$ (left). We assume $\alpha, \beta, \gamma \in [0, \pi]$.

cays into πK and $\pi\pi$ final states, leaving a detailed discussion to a subsequent publication. Here we have focused on robust predictions for ratios of CP-averaged decay rates. Our result for the ratio of the $B \rightarrow \pi^+\pi^-$ and $B \rightarrow \pi^\mp K^\pm$ decay rates is in disagreement with the current experimental value, unless the weak phase γ were significantly larger than 90° .

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